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Curriculum and Culture: An Exploratory Examination of Mathematics Curriculum Materials in Their System and Cultural Contexts

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Abstract: This paper discusses existing cross-system comparisons in mathematics curriculum, by relating the examination of mathematics curriculum materials to their system and cultural contexts. Specifically, this paper elaborates the system and cultural contexts of four education systems: the United States, Hong Kong, Mainland China, and Singapore. By designating a common content topic (i.e., algebra), this paper provides contextual explanations of cross-system similarities and differences in structuring and presenting algebra content in nine eighth-grade mathematics textbooks. Such a contextualization shows the promise of furthering our understanding of cross-system similarities and differences in mathematics curriculum materials and also provides a basis for justifying the possibilities and constraints in learning from cross-system studies of curriculum.

Key words: Mathematics curriculum; Cultural contexts; Cross-system study; School algebra

Introduction

Curriculum[1], as specified in different education systems, outlines students' learning experiences in classrooms. Existing cross-system studies have often focused on examining the similarities and differences in mathematics curriculum and its potential impact on students' mathematics achievement (e.g., Fuson, Stigler, & Bartsch, 1988; Li, 2000; Schmidt et al., 2001; Westbury, 1992). Relevant findings have been informative for understanding students' performance similarities and differences. However, there is often lack of research effort in examining the relationships between curriculum variations and their system and cultural contexts. Without a consideration of system and cultural contexts, it is difficult to understand and adapt certain approach(s) from one education system for making successful educational changes in another (Ginsburg, Leinwand, Anstrom, & Pollock, 2005; Ginsburg, Cooper, Ragbu, & Zegarra, 1990; Li & Ginsburg, 2006). In the case of adopting Singapore mathematics textbooks in the U.S. schools, Ginsburg, Leinwand, Anstrom, and Pollock (2005) found that a successful adoption of Singapore textbooks takes much more than simply using the textbooks. Thus, to further our understanding of the similarities and differences of mathematics curriculum materials from different education systems, it is important to situate variations in mathematics curriculum into their system contexts. Specifically, the purpose of this paper is to relate system context with cross-system similarities and

differences in mathematics curriculum materials from the United States and some other education systems in East Asia.

A recent study analyzed the inclusion and presentation of algebra content in nine mathematics textbooks for the eighth grade from four education systems: the United States, Hong Kong, Mainland China, and Singapore (Li, 1999). The selection of these four education systems in that study provided a basis for examining the relationships between social-cultural characteristics and curriculum variations. In particular, Li and Ginsburg (2006) explored content variations, across these four systems, in knowledge selection and organization envisioned in the nine textbooks and their relationships with the social-cultural norms concerning authority in each society. Our findings suggest the feasibility and value for further studies to examine subtle differences embedded in curriculum that reflect social-cultural differences. Thus, this paper aims to extend previous studies to illustrate the relationship between system contexts and curriculum variations in structuring and presenting algebra content in mathematics textbooks from the United States and the three Asian education systems (i.e., Hong Kong, Mainland China, and Singapore).

Characteristics of Selected Education Systems and Their Cultural Contexts

Although the four selected education systems share some broad similarities in student schooling and mathematics education (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997), they differ in many aspects, such as geographic locations, economy, culture, political system, educational administration and student mathematics achievement. Some of these differences may not relate directly to curricular variations. For example, the Third International Mathematics and Science Study (TIMSS) curriculum study investigated whether regional country groupings have shared visions in curricular intention (see Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). TIMSS analyses indicated that there were many more differences than similarities in curricular intention among these country groupings. Furthermore, there were no substantial similarities when the participating countries were grouped in terms of their economy as measured by a per capita gross national product (GNP). However, some other cross-system differences may have a close relationship to mathematics curriculum. In particular, curriculum, as part of an education system, is inherently influenced by the differences in system structures and cultures (e.g., Griffiths & Howson, 1974; Romberg, 1992) and is related to student achievement (e.g., Westbury, 1992; Schmidt et al., 2001). For the purpose of discussion in this paper, the following highlights the characteristics of the four education systems and their cultural contexts.

Characteristics of the selected Asian education systems and their cultural contexts

Hong Kong, Mainland China, and Singapore all have a centralized education system, and they differ from the United States that has a decentralized education system (Beaton et al., 1996)[2]. In particular, curriculum guides and textbooks used in these three Asian education systems are required to bear an approval from a system-level authority. In contrast, the United States leaves such responsibilities of developing curriculum guides and selecting textbooks to states, local school districts, individual schools, or even individual teachers.

Moreover, Hong Kong, Mainland China, and Singapore share the same culture root. Although there are a variety of ethnic groups living in Hong Kong, Mainland China, and Singapore, the majority of their populations are Chinese (see Brimer, 1988; Dong, 1988; Thomas, 1988). Some education researchers even argued that there is a "Confucian Heritage Culture" (CHC) in this region (e.g., Bond, 1996; Wong, 2004). Confucius is a legendary figure in the Chinese history, whose thoughts in morals, government, and education have been very influential in forming Chinese culture. In particular, there are some salient values, as summarized by Bond (1996) from relevant studies, common to the CHC regions of Hong Kong, Singapore and Mainland China. Such common values include hierarchy, discipline and a strong achievement orientation.

Confucius stressed the role of education in developing people's moral superiority that forms the basis of leadership and governance. The emphasis on moral development through education in Confucianism, however, had negative impact on learning mathematics in ancient China. Basically, in formal schooling in ancient China, "the writing of examination essays became almost the exclusive concern of all who had reached the tertiary level of formal education, and collected essays by successful candidates in previous examinations, appearing in ever increasing volume and variety, formed the single most important body of literature for all aspiring scholars." (Hu, 1984, p. 12). Whereas, practical knowledge and skills were treated as incompatible for fostering moral superiority. Specifically, "arithmetic had been scorned as belonging to shopkeepers, …" (Hu, 1984, p. 20). Therefore, mathematics was not an important or even a necessary subject in ancient Chinese education. Although mathematics has become one of the primary school subjects for all students in modern China, what is valued through learning mathematics certainly bears its cultural tradition. As indicated by Stigler and Perry (1988), mathematics curriculum and teaching in China tended to focus on students' acquisition of abstract content knowledge that is different from the case of the United States. Taken together, learning mathematics in the CHC culture has been valued more for

acquiring abstract content knowledge and developing students' thinking than for developing students' skills of solving practical problems in everyday life.

Although Hong Kong, Mainland China, and Singapore share similar culture roots, they differ in that Hong Kong and Singapore were both British colonial territories (Brimer, 1988; Thomas, 1988). Therefore, English is also an official language in both Hong Kong and Singapore, especially for government and trade. Although Hong Kong began to adopt Mandarin as its official language with the return of sovereignty to China in 1997, the change may take a long time (Adamson & Lai, 1997). Because of the role played by the English language and because of the influence of non-Confucian (i.e., Western) culture, the textbooks from Hong Kong and Singapore may be somewhat more like the ones used in the United States than is the case for Mainland China's textbooks. Therefore, the selection of these three Asian education systems presumably provides a basis to examine possible variations in terms of the relationships between mathematics textbooks and their system and cultural contexts when compared to the United States.

Characteristics of the United States education system and its cultural context

Relative to the CHC culture, the United States does not have a long history. As an immigrant country, this nation has the strength of diversity and creation of new ideas. Education has generally been required as a necessary and important preparation for qualified workers for developing economic productivity. Efforts to understand students and their learning process are well rooted in Western culture (e.g., Dewey, 1956) and have led to the generation and application of many perspectives and approaches for educational practice. For example, one popular perspective about students' learning of mathematics nowadays is constructivism, which emphasizes the process of students' own construction of mathematics knowledge through their active participations in problem-solving activities. This perspective, different from some other perspectives (e.g., drill-and-practice), focuses on students' cognition involved in learning. Because one perspective can be advantageous in some aspects but may not be in others, educators may differ in their choices of educational perspectives or approaches and may combine different perspectives to pursue better educational effects in their practices. However, for the majority of mathematics teachers, it is often too overwhelming to understand and use several different educational perspectives and approaches in classroom instruction.

Because of its wide applicability, mathematics is taken as a necessary and important training for every student in the United States (National Council of Teachers of Mathematics [NCTM], 1989, 2000). Specifically, learning mathematics has been

advocated with a strong orientation to meet industrial needs and society development (NCTM, 1989). Problem solving is suggested as the focus of school mathematics. And because the nation has a decentralized education system, there is no one common vision on what mathematics content knowledge is important for school students across the United States. Thus, the value of learning school mathematics has often placed more on developing students' skills of solving practical problems than on acquiring abstract knowledge or developing students' mathematics thinking. In fact, it is common in the United States that students do not take mathematics learning seriously. This value orientation is based on a consideration on mathematical utility and it is certainly different from the one embedded in the CHC culture.

Although the United States enjoys the variety of cultural heritage brought into the country by different immigrants, the majority of its population is the people whose ancestors had emigrated from Europe. Thus, the US carries its own cultural characters when compared to the CHC culture. For example, the official language used in US school education is English. The difference in language use, as argued by many researchers (e.g., Miller, Smith, Zhu, & Zhang, 1995; Miura, Kim, Chang, & Okamoto, 1988), has influenced students' early learning of a base-10 number naming system. Some researchers have even extended their cross-system investigations between the US and some Asian education systems as cross-cultural studies that relate students' mathematics performance with some cultural factors, such as parents' belief and teacher pedagogical approaches (e.g., Cai, 2005; Stigler & Perry, 1988). However, there is a lack of systematic investigation on mathematics curriculum materials as situated within specific systems and cultures that compares the United States and some Asian education systems (Li & Ginsburg, 2006), although similar studies might have been carried out elsewhere, such as comparing mathematical values conveyed in Australian and Chinese textbooks (Cao, Seah & Bishop, 2006).

Characteristics of Algebra Content Inclusion and Presentation in Mathematics Textbooks from the Four Education Systems

Materials

Nine mathematics textbooks for eighth grade from four education systems were analyzed in a previous study (Li, 1999). These nine textbooks are the ones that were also examined in the TIMSS curriculum study (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). Among these nine textbooks, five textbooks were from the US, one from Hong Kong, two from Mainland China, and one from Singapore. Among the five US textbooks, one was an algebra-specific textbook (hereafter, called US-A) and four others were popular non-algebra-specific mathematics

textbooks for the eighth grade (hereafter, called US-NAS). Focusing on the algebra content, the previous study examined these selected textbooks in details with respect to three aspects: the inclusion of algebra content, the ways of organizing and presenting algebra content, and the to-be-solved problems provided for students' practice. The following sections present a summary about the findings on textbooks' similarities and differences in algebraic content inclusion, presentation and organization.

General similarities and differences of textbooks from the four education systems

Many similarities and differences exist among the textbooks from the four education systems. One of the most obvious differences among these textbooks is the appearance of the books and the general way in which the material is presented. For example, all the textbooks, except for the ones from Mainland China, were designed as a single volume with large page size to be used throughout the whole eighth grade. In Mainland China, two textbooks with much smaller page size were produced as content specific volumes for eighth grade; one is titled “algebra”, and the other “geometry”. These two books can be used either separately or in combination in each of two semesters. Moreover, only the textbooks from Mainland China are written in Chinese. All others are written in English. The textbook from Hong Kong includes a few Chinese terms that are translated for the mathematics concepts defined and explained in the text. Except for the number of textbook volume, page size and language use where the textbooks from Mainland China distinguish themselves from other textbooks, many other differences can be noted between the US textbooks and the ones from the three education systems in East Asia.

First, the US textbooks are longer than those published in East Asia, and all are hard cover books. The average number of pages in the five American textbooks is 622. The fewest number of pages in the US textbooks is 546 pages, and the greatest number is 760 pages. Asian textbooks are much shorter, and they are paperback books; none has more than 415 pages. The longest one from Hong Kong has 415 pages. The shortest one from Mainland China has 228 pages. The Asian textbooks tend to be tersely written, while the US textbooks often contain various topics and problems that make the books thicker than the ones from East Asia.

Second, the US textbooks are colorful, with illustrations and figures on nearly every page. Moreover, except for the US algebra textbook, the other four US textbooks tend to organize content in a form of two-page lesson units and to include mathematics problems given in the context of everyday life. The use of small content sections with detailed explanation in the texts shows the tendency of

decreasing mathematics content difficulty. In contrast, the Asian textbooks are basically black-and-white and include few illustrations. The content is organized in large content sections that tend to maintain the coherence of mathematical knowledge. The content is presented in brief discussions that emphasize the abstract nature of mathematical concepts.

Finally, the American textbooks appear to have more repetition and review in the presentation of materials. The textbooks tend to include problems that are related to many different topics, such as geometry, addition, and measurement, and that serve many different purposes such as skill review. In fact, except for the US algebra textbook, it is difficult to identify the grade-level appropriateness of the other four US-NAS textbooks. The Asian textbooks, in contrast, seem to be developed on the assumption that knowledge should be cumulative from semester to semester. Because of the inclusion of more advanced content topics in the Asian textbooks, it is easier to tell their grade level than the US textbooks.

Similarities and differences in algebraic content inclusion in textbooks from the four education systems

More differences than similarities were found between US and Asian textbooks in their inclusion of mathematics content topics (e.g., Schmidt, McKnight, Valverde, Houang, & Wiley, 1997) and algebra content for the eighth graders in specific (Li, 1999). Although all the textbooks selected from the four education systems include algebraic content for eighth graders, their inclusions of algebraic content vary in many ways. Across all the textbooks, there are striking differences in the algebraic topics included. In particular, the algebraic topics included in the Asian textbooks are all advanced ones whereas the US-NAS textbooks include only elementary topics and the US-A textbook covers a broad range of topics from elementary to advanced ones. The similarities in content requirements and emphases exist among the Asian textbooks but not among the five US textbooks, nor between the Asian and the US textbooks (see Table 1 on next page).

Similar to what has been found in the TIMSS curriculum study (Schmidt, McKnight, & Raizen, 1997), the US textbooks present a diverse vision on what content needs to be included for students to learn. Specifically, the US has the textbook (US-A text) with the heaviest emphasis on algebra and some others (four US-NAS texts) with the lightest emphasis on algebra when compared with the textbooks from the three Asian education systems. The differences between these two types of US textbooks are striking. Moreover, the diversity is also evidenced across individual US textbooks. For each of the US textbooks examined in the study

Table 1
Textbook Variations in Algebraic Topic Inclusion, Content Requirement and Emphasis

	Topic Inclusion	Content Requirement	Content Emphasis
Asian texts	advanced topics	high	29%-50% chapters in algebra
US-A text	a broad range of topics	a broad range	83% chapters in algebra
US-NAS texts	elementary topics	low	13%-21% chapters in algebra

the algebraic chapters also contain non-algebraic topics, which is in contrast to the consistent inclusion of algebraic topics in the algebraic chapters of the Asian textbooks. The inclusion of different topics in algebraic chapters in the US textbooks shows diverse approaches for including and organizing mathematics content. In contrast, the Asian textbooks are similar in their content emphasis and requirements on algebra and show a focused approach in organizing content in their algebraic chapters.

Similarities and differences in algebraic content presentation and organization in textbooks from the four education systems

Although the US textbooks vary widely in their inclusion of algebraic content topics, these textbooks share many similarities in presenting and organizing mathematical content. In particular, content organization style across all nine textbooks is remarkably consistent within a culture and is different across cultures. That is, the US textbooks examined in the previous study (Li, 1999) present similar features of content organization, and these features are distinctly different from those found in the Asian textbooks, which are quite similar to each other. Specifically, the five US textbooks are similar in their ways of organizing content within a chapter. The content in a chapter of the US textbooks is organized with various categories of small content sections. The inclusion of various categories of

content sections shows that the US textbooks value the use of different instructional activities rather than solely content introduction. However, the use of many small content sections fragments the instructional content into small and different pieces, which facilitates the formation of an "assembly line" of content organization. The picture of content organization presented in the US textbooks is thus consistent with what has been found in the TIMSS curriculum study based on content topic coverage at different grade levels (Schmidt, McKnight, & Raizen, 1997). As interpreted by Schmidt, McKnight, and Raizen (1997), US school mathematics has a long history of fragmenting content and gradually building up central topics. In theory, the difficulty of learning mathematics can be decreased through breaking mathematics content into small pieces. However, the diverse organization of various content sections evidenced in the US textbooks presents a loosely structured content, which in fact poses a challenge for teachers and students to build up central topics from small pieces.

On the contrary, the textbooks from the three education systems in East Asia use few categories but larger content sections to organize content within their algebraic chapters. Because the majority of content sections are the ones on content introduction, these Asian textbooks show a concentration on content instruction other than on providing various activities and to-be-solved problems for students' practice. The use of large content sections further helps to make in-depth content introduction in these textbooks. Moreover, each of these Asian textbooks organizes its content sections with a distinct pattern in algebraic chapters. The regularized organization of content sections thus has the potential to facilitate students' structuring of mathematical content when they are learning from the mathematics textbooks.

If the organization of content sections within algebraic chapters only reflects general organization features, the results on content presentation features within content sections can illustrate textbook style developed for teaching and learning algebra. Specifically, the Asian textbooks present instructional content in a concise and logical manner, in which mathematics knowledge is always placed at the beginning of a content section, followed by several worked examples and then by a few sets of problems for student practice. Because the content is often presented without the specifications of how it can be developed from a problem context, the textbooks place heavy reliance on the teacher to assist students with discussion and elaboration of the content. This way of content presentation reflects a clear intention of "explaining" knowledge prior to application. It shows a traditional style in presenting mathematical content that can be found in many mathematics textbooks decades ago. The US-A textbook is similar to the Asian textbooks in taking the traditional style for presenting algebraic content within content sections. Moreover,

the Asian textbooks use more page space for content introduction than that for to-be-solved problem sets, which is just opposite to the US-A textbook.

Different from the traditional style of content presentation noted in the Asian textbooks, the four US-NAS textbooks present their instructional content in a more dynamic way. Specifically, the introduction of mathematics knowledge in the US-NAS textbooks can be found as placed either before worked example(s), within a worked example, or after a worked example. Many to-be-solved problems are then given to serve several practice purposes. The textbooks tend to introduce mathematics knowledge within an illustrative context. This way of content presentation shows an emphasis on linking mathematical knowledge with the real world, which reflects a problem-based style for teaching and learning algebra. In this style, students are expected to explore mathematical ideas within a problem context. Students' construction of mathematics knowledge from a problem context appears to be emphasized in these textbooks. Moreover, the US textbooks tend to present content information step-by-step and in details. Great efforts have been put into the writing of these textbooks to make the content easily understandable for the learners. After content presentation, all five US textbooks tend to include many to-be-solved problems with great variations in problems' requirements for students' practice.

Taken together, the US textbooks tend to dissemble content into small content sections, introduce content within real-world problem contexts, emphasize students' practice, and include various to-be-solved problems to develop students' mathematical problem solving competency. In contrast, the Asian textbooks are similar in their content emphasis and their high mathematics requirements in algebra. They tend to include large content sections of coherent algebra topics, introduce content as pure mathematics, and include to-be-solved problems for strengthening students' acquisition of newly taught content knowledge and procedural skills.

Understanding Similarities and Differences of Mathematics Curriculum Materials in Their Cultural Contexts

Content inclusion and cultural values

The within-cultural similarities and cross-cultural differences in algebra content inclusion in eighth-grade mathematics textbooks show a close relationship between mathematics curriculum and characteristics of cultural contexts. Specifically, Hong Kong, Mainland China and Singapore are all centralized education systems and share the same cultural roots. Textbooks' content requirements and emphasis in algebra present a coherent outline for eighth graders from the three Asian education

systems. Moreover, the CHC culture values more on students' learning of abstract mathematics knowledge than on developing their skills of solving practical problems in everyday life. Correspondingly, these Asian textbooks tend to include advanced and consistent algebra topics other than practical problems that require the use of mathematics knowledge.

In contrast, textbook writing and publication form a market-based business in the United States. Commercially produced textbooks compete to accommodate the diverse market (Schmidt, McKnight, & Raizen, 1997). The lack of a common curriculum guideline at the national level likely facilitates the development of such diversity in curriculum requirements. Thus, although the five selected US textbooks are the ones produced within an education system, they can be similar in some ways but strikingly different in others. Moreover, because students' learning of mathematics in the United States is valued for developing their problem-solving competence, textbooks were written with an emphasis on including various mathematics problems (also see Zhu & Fan, 2006) other than increasing content requirements in mathematics. In fact, the US textbooks tend to decrease mathematics content requirements through including elementary topics and fragmenting a content topic into small pieces. The cross-cultural variations in algebraic content requirements and emphasis reflect different cultural values in shaping what is possible and important for students to learn.

The similarities and differences in algebra content inclusions mirror the strength and weakness of curriculum materials in both the three Asian education systems and the US system. In particular, the Asian textbooks consistently provide students with opportunities to learn advanced algebra content. Curriculum materials show a focused approach in including and organizing advanced content in their algebraic chapters. This approach could be seen as a disadvantage as it provides very limited options to meet diverse students' needs and pays limited attention to developing students' problem-solving competence. But with the support of the CHC culture, this approach seemingly works well in developing culturally valued students' competence in mathematics in the three Asian education systems. Comparatively, curriculum diversity in the US textbooks could be a curricular strength as various curriculum materials provide options to meet the needs of a diverse student population. However, without a common vision of what students should learn in algebra, curriculum diversity may present less positive effects than what one may expect. In particular, because the US-NAS textbooks are more popular than the US-A textbook in the system, the results indicate that most US eighth graders had very limited opportunity to learn algebra other than elementary algebra content. The inclusion of generally low demands for students' learning of algebra with the non-algebra-specific textbooks certainly needs to be changed. However, curriculum

changes may not simply be as adding advanced algebra topics in mathematics textbooks for all US eighth graders. As a matter of fact, simply adding advanced algebra content in curriculum materials is unlikely supported in the US system and its cultural context. As suggested by many US mathematics educators (e.g., Silver, 1995; Steen, 1992), a change is needed to develop students' algebraic thinking and reasoning other than simply adding advanced algebra topics. Such an approach places more emphasis on content connections, breath, and cognitive requirements than on the depth of mathematics content. For example, several newly developed curricular materials in the United States, all funded by National Science Foundation (NSF), have been designed with focal mathematics ideas, demanding tasks, and integrated mathematics topics (Reys, Robinson, Sconiers, & Mark, 1999). Students' opportunity to access algebraic ideas is expanded through solving challenging mathematics tasks in many different units rather than determined by counting algebraic topics included. In contrast, the Asian textbooks place less emphasis on content connections but more on the depth of specific content topics. The America's approach likely suits the US cultural context and certainly differs from what has been envisioned in the Asian textbooks.

Content presentation and organization and cultural expectations for teachers' teaching and students' learning

Although the ways of content presentation and organization in textbooks are not the same as what happened in classroom instruction, textbooks' styles in presenting content could certainly reflect cultural expectations for the teaching and learning of mathematics. In particular, the textbooks from the three selected Asian education systems share similar style in presenting and organizing algebra content and differ from many other US textbooks. The within-cultural similarities and cross-cultural differences do suggest cultural influences on each system's expectations for the teaching and learning of mathematics.

The traditional style adopted in the Asian textbooks and the US algebra textbook shows a clear, logic, and consistent presentation and organization of content that makes the textbooks feasible for students' reading or content review. With the presentation of mathematical content at the beginning of a unit, students can easily understand what needs to be learned. The follow-up inclusion of several worked-out examples enables students to learn how the newly introduced mathematics content can be used. Few sets of to-be-solved problems further provide students opportunities to practice and apply what has been introduced in the text. Therefore, the textbook can serve as a resource book for students' learning of mathematics. However, because content presented in these textbooks is tersely written, to develop a good understanding of mathematics content requires students' great efforts and

reflective thinking when learning from the textbook. Moreover, the textbook has very limited variations. It is therefore not suitable for a direct adoption for day-to-day classroom instruction. Consequently, this approach would pose challenges to mathematics teachers to structure their classroom teaching in dynamic and effective ways. Thus, the textbooks reflect cultural expectations in students' efforts and teacher's guiding role in the process of learning mathematics from the textbooks.

In the four US-NAS textbooks, the content is fragmented and presented in great detail. This way of content organization and presentation decreases mathematics content difficulty and thus makes it easier for students to learn mathematics by themselves. At the same time, it also means to require less effort from students to learn mathematics than what effort requirement might place on students by the traditional style textbooks. This reflects a general belief in the US culture that successful learning of mathematics depends more on individual intelligence than on efforts (Hess & Azuma, 1991; Stigler & Perry, 1988). Moreover, because mathematics content is presented frequently in illustrative problem contexts rather than simply stated, the problem-based style evidenced in the US-NAS textbooks presents a ready-to-be-used format of content presentation for classroom teaching. The style reflects a cautious and thoughtful design of content presentation and is consistent with what has been broadly suggested for teaching mathematics in classrooms (e.g., Silver, Kilpatrick, & Schlesinger, 1990). It also embodies the intention of developing students' inquiry in the process of learning mathematics. However, the teacher's role is unspecified in this style. Furthermore, the US textbooks often lack a summary of mathematical content being introduced in a content section or a chapter, which likely increases the difficulty of making effective use of the textbooks. That is, as the US-NAS textbooks place much attention on students' knowledge construction process in their content presentation, they lack clear indications of where the knowledge construction process is leading.

The above two styles of textbooks' content presentation show both possible advantages and disadvantages in influencing the teaching and learning of mathematics. They also reflect different cultural perspectives for the teaching and learning of mathematics. Specifically, the US textbooks include both styles of content presentation (i.e., the problem-based style in the four US-NAS textbooks and the traditional style in the US-A textbook), which illustrates the diversity of perspectives and approaches for the teaching and learning mathematics in the US education system. Because four US-NAS textbooks adopt the problem-based style, it is certainly more popular than the traditional style and reflects the common belief that students' learning is a knowledge construction process. However, the problem-based style tends to develop students' inquiry but not necessarily reflective thinking and generalization in their learning of mathematics. The teacher's role is also not

emphasized. In contrast, all the textbooks from the three Asian education systems adopt the traditional style and rely on students and teacher's efforts in learning from the textbooks. This style asks for teachers' guidance and students' reflective thinking but not an inquiry mode at the beginning of mathematics learning process. Taken together, these differences suggest internal connections between cultural expectations and textbook content presentation. Thus, while both the US textbooks and the Asian textbooks can learn from each other to develop content presentation and organization, what needs to be changed in one textbook may not simply be what is missing in this textbook but available in another. A thoughtful improvement of textbook content presentation and organization needs to go beyond finding cross-system similarities and differences in mathematics curriculum.

Coda

This paper aims to connect the examination of mathematics curriculum materials with their system and cultural contexts. In particular, this paper provides contextual explanations of cross-system similarities and differences in structuring and presenting algebra content in the selected mathematics textbooks from the United States and the three Asian education systems (i.e., Hong Kong, Mainland China, and Singapore). Such a contextualization shows the promise of expanding our understanding of cross-system similarities and differences in mathematics curriculum, which has often been related to students' mathematics performance. However, as discussed in this paper, variations in curriculum materials cannot simply be taken as the only basis for suggesting changes. Any thoughtful changes in curriculum materials require a further understanding and consideration of their system and cultural contexts. Moreover, because teacher and students are the persons who will finally determine how curriculum materials are used and thus show their effects, cross-system exploration of teacher and students' use of textbooks then becomes necessary to further our understanding of possible cultural variations. In this way, the development of cross-system study on curriculum materials and their uses may eventually lead to the improvement of teaching and learning of mathematics in classrooms.

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Notes:

[1] Curriculum is a broad concept that provides the fundamental structure of students' learning experiences in school. This paper only considered curriculum in its intended aspects that help to structure and guide students' experiences in school. The intentions and expectations are articulated by policy makers and curriculum material writers and presented as guidelines and curriculum materials at various detailed levels for school education.

[2] For more information and related discussion about the US education system and the three education systems in East Asia, see Li and Ginsburg (2006), Usiskin and Dossey (2004), and Zhu and Fan (2006).

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